NATIONAL SENIOR CERTIFICATE/
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* **12**

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2021

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/PUNTE: 150

These marking guidelines consist of 24 pages.
Hierdie nasienriglyne bestaan uit 24 bladsye.

NOTE:
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:
- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

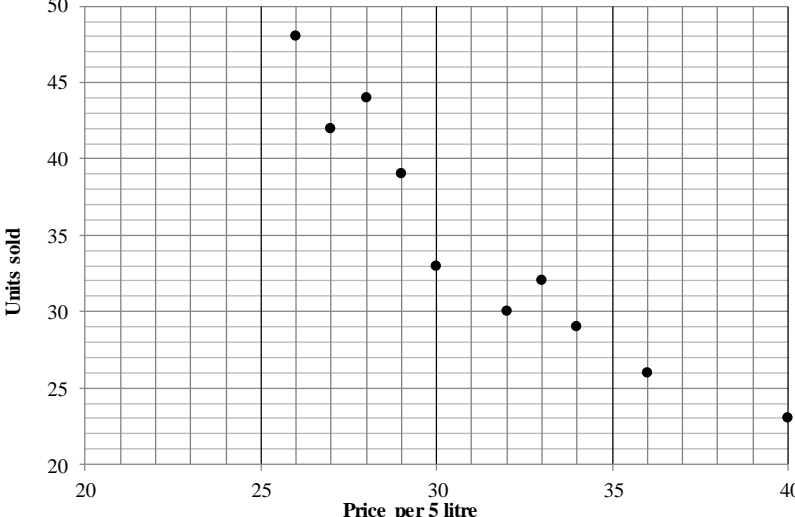| GEOMETRY ● *MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering *('n Punt vir 'n bewering is onafhanklik van die rede)*
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede *('n Punt word slegs vir die rede toegeken as die bewering korrek is)*
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/*VRAAG* 1

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

1.1.1	$\bar{x} = \dfrac{396}{18}$ $\qquad$ Answer only: Full marks / *Slegs antw: Volpunte* $\bar{x} = 22$	✓ 396 ✓ answer (2)
1.1.2	$\sigma = 10{,}1707 \approx 10{,}17$	✓ answer (1)
1.1.3	$\bar{x} + \sigma = 32{,}17$ $\therefore 5$ days	✓ 32,17 ✓ 5 (2)
1.2	$22 \times 18 = 396$ ordered/*bestel* $20 \times 18 = 360$ sold/*verkoop* Total not sold/*Totaal nie verkoop nie*: 36 **OR/*OF*** $22 - 20 = 2$ $2 \times 18 = 36$	✓ $18\bar{x}_1$ and $18\bar{x}_2$ ✓ answer (2) ✓ $\bar{x}_1 - \bar{x}_2$ ✓ answer (2)
1.3.1	Option B/*Opsie B* <u>Any one of the following reasons/*Enige een van die vlg redes*</u>: - Median/*Mediaan* = 18,5 $Q_1 = 14$ - IQR = 21 - Mean > Median, therefore the data is skewed to the right	✓ B ✓ reason (2)
1.3.2	Data is positively skewed/skewed to the right *Data is positief skeef/skeef na regs*	✓ answer (1)
		[10]

QUESTION/*VRAAG* 2

Price of milk in rands per 5-litre container (x) *Prys van melk in rand, per 5 liter-houer* (x)	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y) *Aantal 5 liter-houers melk verkoop* (y)	48	30	26	44	23	32	39	29	42	33

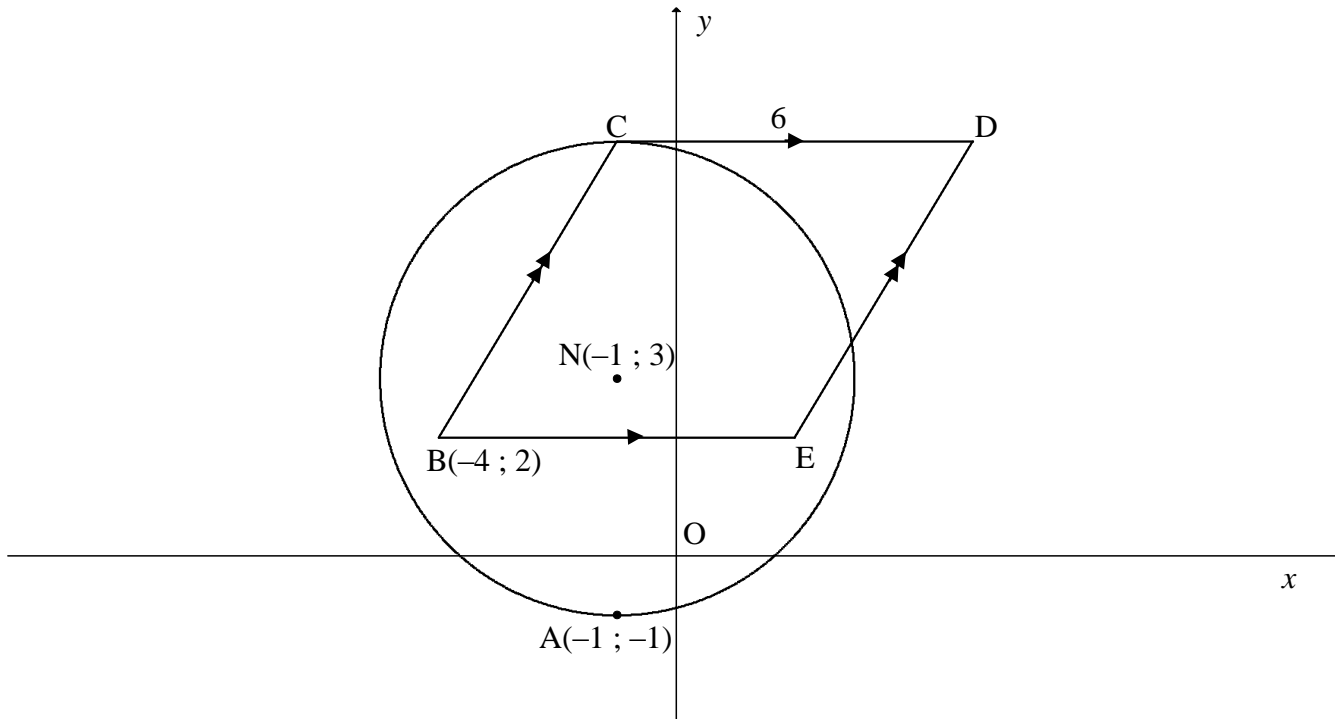| 2.1		
		1 mark: 3 to 5 points plotted correctly 2 marks: 6 to 9 points plotted correctly 3 marks: all points plotted correctly (3)
2.2	$a = 90,478... \approx 90,48$ $b = -1,773... \approx -1,77$ $\hat{y} = 90,48 - 1,77x$ Answer only: Full marks *Slegs antw: Volpunte*	✓ a ✓ b ✓ equation (3)
2.3	$y = 23,069… \approx 23,07$ units/*eenhede* (calculator/*sakrekenaar*) **OR/*OF*** $y = 90,48 - 1,77(38)$ $y = 23,22$ units/*eenhede*	✓✓ answer (2) ✓ substitution ✓ answer (2)
2.4	$r = -0,94$ The value of *r* indicates a strong relationship between the cost per 5 litre and the number of units sold ∴ there is a good chance of the prediction being accurate./ *Die waarde van r dui 'n sterk vewantskap tussen die koste per 5 liter en die aantal eenhede verkoop aan* ∴ *daar is 'n goeie kans dat die voorspelling akkuraat is*	✓ value of *r* **OR/*OF*** strong relationship/*sterk verwantskap* ✓ accurate/*akkuraat* (2)
		[10]

QUESTION/*VRAAG* 3

3.1.1	$m_{BE} = m_{CE} = \dfrac{0-(-2)}{12-4}$ **OR/*OF*** $m_{BE} = m_{CE} = \dfrac{-2-0}{4-12}$ $\qquad = \dfrac{1}{4} \qquad\qquad\qquad\qquad = \dfrac{1}{4}$	✓ substitution C & E ✓ answer (2)
3.1.2	$m_{AB} = \tan 81,87°$ $m_{AB} = 7$ |Answer only: Full marks *Slegs antw: Volpunte*|	✓ substitution ✓ answer (2)
3.2	$y = mx + c \qquad\qquad y - y_1 = m(x - x_1)$ $0 = \dfrac{1}{4}(12) + c \qquad$ or $\quad y - 0 = \dfrac{1}{4}(x - 12)$ $c = -3 \qquad\qquad\qquad y = \dfrac{1}{4}x - 3$ $y = \dfrac{1}{4}x - 3$	✓ substitution of E ✓ answer (2)
	OR/*OF* $y = mx + c \qquad\qquad y - y_1 = m(x - x_1)$ $-2 = \dfrac{1}{4}(4) + c \qquad$ or $\quad y - (-2) = \dfrac{1}{4}(x - 4)$ $c = -3 \qquad\qquad\qquad y = \dfrac{1}{4}x - 3$ $y = \dfrac{1}{4}x - 3$	✓ substitution of C ✓ answer (2)

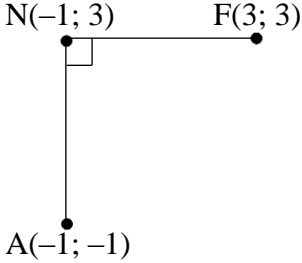| 3.3.1	$y = \frac{1}{4}x - 3$	
	$k = \frac{1}{4}k - 3$	✓ substitution
	$\frac{3}{4}k = -3$	
	$k = -4$	✓ answer
	$\therefore B(-4; -4)$	(2)

OR/*OF*

$m_{BE} = \frac{1}{4}$ **OR/*OF*** $m_{BE} = \frac{1}{4}$ ✓ substitution

$\frac{0-k}{12-k} = \frac{1}{4}$ $\frac{k}{k-12} = \frac{1}{4}$

$-4k = 12 - k$ $4k = k - 12$

$k = -4$ $k = -4$ ✓ answer

$\therefore B(-4; -4)$ (2)

OR/*OF*

$m_{AB} = \tan 81,87°$

$m_{AB} = 7$

                                                 ✓ substitution

$m_{AB} = \frac{10-k}{-2-k}$

$7(-2-k) = 10 - k$

$-14 - 7k = 10 - k$

$-6k = 24$

$k = -4$ ✓ answer

$\therefore B(-4; -4)$ (2)

OR/*OF*

EB: $y = \frac{1}{4}x - 3$ and   AB: $y = 7x + 24$ ✓ equating EB & AB

$\frac{1}{4}x - 3 = 7x + 24$

$\frac{27}{4}x = -27$

$x = k = -4$ ✓ answer

$\therefore B(-4; -4)$ (2)

3.3.2	In $\triangle AFG$: $m_{AC} = \dfrac{10-(-2)}{-2-4} = -2$ $\tan\theta = m_{AC} = -2$ $\theta = 180° - 63,43...°$ $\therefore \theta = 116,57°$ $\therefore \hat{A} = 116,57° - 81,87°$ [ext $\angle$ of $\triangle$] $\therefore \hat{A} = 34,70°$	✓ $m_{AC} = -2$ ✓ $\tan\theta = -2$ ✓ $\theta = 116,57°$ ✓ answer (4)
	OR/*OF* In $\triangle ABC$: $\quad a = BC = 2\sqrt{17}; b = AC = 6\sqrt{5}; c = AB = 10\sqrt{2}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\left(2\sqrt{17}\right)^2 = \left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - 2(6\sqrt{5})(10\sqrt{2}).\cos A$ $\cos A = \dfrac{\left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - \left(2\sqrt{17}\right)^2}{2\left(6\sqrt{5}\right)\left(10\sqrt{2}\right)}$ $\quad = 0,822...$ $\quad\therefore A = 34,7°$	✓ all 3 lengths ✓ substitution into the correct cosine rule ✓ cos A subject ✓ answer (4)
3.3.3	$M\left(\dfrac{12+(-2)}{2} ; \dfrac{10+(0)}{2}\right)$ Diagonals intersect at the point (5 ; 5)	✓ *x*-value ✓ *y*-value (2)
3.4.1	$BE = ET$ $4\sqrt{17} = \sqrt{(12-p)^2 + (0-p)^2}$ $\left(4\sqrt{17}\right)^2 = \left(\sqrt{(12-p)^2 + (0-p)^2}\right)^2$ $272 = 144 - 24p + p^2 + p^2$ $p^2 - 12p - 64 = 0$ $(p-16)(p+4) = 0$ $\therefore p = 16 \quad$ or $\quad p = -4$ (n.a.) $\therefore T(16; 16)$	✓ substitution of E & T ✓ equating ✓ standard form ✓ factors ✓ $p = 16$ (5)
3.4.2a	$(x-12)^2 + y^2 = \left(4\sqrt{17}\right)^2 = 272$	✓ LHS ✓ RHS (2)
3.4.2b	$m_{radius} = \dfrac{1}{4}$ $m_{tangent} = -4$ $y = -4x + c \quad$ **OR/*OF*** $\quad y - y_1 = -4(x - x_1)$ $-4 = -4(-4) + c \qquad\qquad y - (-4) = -4(x - (-4))$ $c = -20$ $y = -4x - 20 \qquad\qquad\qquad\quad y = -4x - 20$	✓ $m_{tangent}$ ✓ substitution of B ✓ equation (3)
		[24]

QUESTION/*VRAAG* 4

4.1	Radius = 4 units/*eenhede*	✓ answer
		(1)
4.2.1	$CD \perp CN$ $\therefore C(-1 \; ; 7)$	✓ x value ✓ y value
		(2)
4.2.2	$CD = 6$ units $\therefore D(5 \; ; 7)$	✓ x value ✓ y value
		(2)
4.2.3	$\perp h = 5\,$ units $DC = 6\,$ units Area $\Delta BCD = \dfrac{1}{2}(6)(5)$ $\qquad\qquad = 15\,$ units^2 **OR/*OF*** $\perp h = 5\,$ units $DC = 6\,$ units Area $\Delta BCD = \dfrac{1}{2}[$Area of $\|^m]$ $\qquad\qquad = \dfrac{1}{2}\big[(5)(6)\big]$ $\qquad\qquad = 15\,$ units^2	✓ $\perp h = 5\,$ units ✓ substitution into Area formula ✓ answer (3) ✓ $\perp h = 5\,$ units ✓ substitution into Area formula ✓ answer (3)

	OR/*OF* Let angle of inclination of BC = α $\tan\alpha = \dfrac{5}{3}$ $\alpha = 59{,}036...°$ $\hat{BCD} = 180° - \alpha$ $\hat{BCD} = 180° - 59{,}036...°$ $\hat{BCD} = 120{,}96°$ Area $\Delta BCD = \dfrac{1}{2}\left(\sqrt{34}\right)(6)\sin120{,}96°$ $\qquad\qquad = 15 \text{ units}^2$	$\checkmark$ $\hat{BCD} = 120{,}96°$ $\checkmark$ substitution into Area rule $\checkmark$ answer (3)
4.3.1	M(3 ; –1) [reflection of N(–1 ; 3) about the line $y = x$] $\therefore MN = \sqrt{(3-(-1))^2 + (-1-3)^2}$ $MN = \sqrt{32} = 4\sqrt{2} = 5{,}66 \text{ units}$	$\checkmark$ coordinates of M (A) $\checkmark$ substitution of M&N $\checkmark$ answer (3)
4.3.2	M(3 ; –1) $m_{MN} = \dfrac{3-(-1)}{-1-3} = -1$ MN: $-1 = -(3) + c$ or $\quad y - 3 = -1(x+1)$ $\qquad c = 2 \qquad\qquad\qquad y - 3 = -x - 1$ $\qquad \therefore y = -x + 2 \qquad\qquad y = -x + 2$ $x = -x + 2$ $2x = 2$ $x = 1$ $\therefore y = 1$ midpoint (1 ; 1) **OR/*OF*** N(–1 ; 3) $y_F = y_N = 3$ Reflected about $y = x$ $\therefore$ F(3 ; 3) midpoint $\left(\dfrac{-1+3}{2}; \dfrac{-1+3}{2}\right) = (1 ; 1)$	 $\checkmark$ equation of MN $\checkmark$ equating AF & MN $\checkmark$ x value $\checkmark$ y value (4) $\checkmark\checkmark$ coordinates of F $\checkmark$ x value $\checkmark$ y value (4)

(In the OR/OF diagram for 4.3.2: N(–1; 3) and F(3; 3) at top with a right angle at N; A(–1; –1) at bottom.)

OR/*OF*

N(−1; 3)   F(3; 3)
•

A(−1; −1)   M(3;−1)
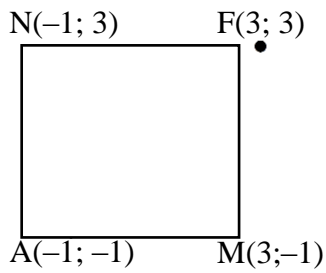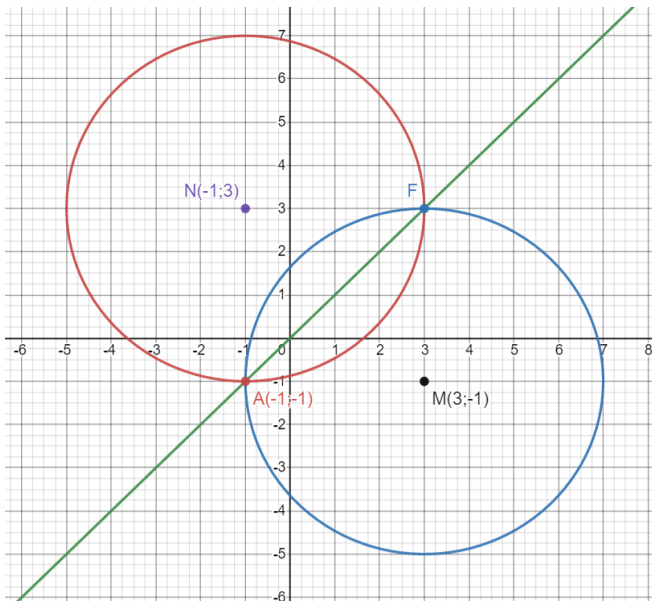
NAMF is a square (NA=NF=AM=MF and NA ⊥ AM)

Midpoint NM = (1 ; 1)
            = Midpoint of AF

✓ NAMF = square
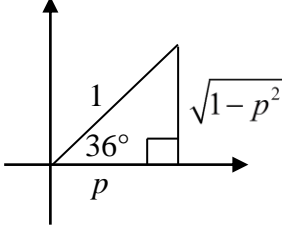
✓x ✓ y of midpt NM
✓ midpt AF

(4)

[15]

QUESTION/*VRAAG* 5

5.1	$\dfrac{\sin 140°.\sin(360° - x)}{\cos 50°.\tan(-x)}$ $= \dfrac{\sin 40°(-\sin x)}{\sin 40°(-\tan x)}$ $= \dfrac{-\sin x}{-\dfrac{\sin x}{\cos x}}$ $= \cos x$	✓ $\sin 40°$ ✓ $-\sin x$ ✓ co-ratio ✓ $-\tan x$ ✓ $\tan x = \dfrac{\sin x}{\cos x}$ ✓ answer (6)
5.2	$\text{LHS} = \dfrac{-2\sin^2 x + \cos x + 1}{1 - \cos(540° - x)}$ $\text{RHS} = 2\cos x - 1$ $\text{LHS} = \dfrac{-2(1 - \cos^2 x) + \cos x + 1}{1 - (-\cos x)}$ $\text{LHS} = \dfrac{-2 + 2\cos^2 x + \cos x + 1}{1 + \cos x}$ $\text{LHS} = \dfrac{2\cos^2 x + \cos x - 1}{1 + \cos x}$ $\text{LHS} = \dfrac{(2\cos x - 1)(\cos x + 1)}{1 + \cos x}$ $\text{LHS} = 2\cos x - 1$ $\therefore \text{LHS} = \text{RHS}$	✓ identity i. t. o. $\cos x$ ✓ $\cos(540° - x) = -\cos x$ ✓ standard form ✓ factors (4)
5.3.1	$\sin 36° = \sqrt{1 - p^2}$ $\tan 36° = \dfrac{\sqrt{1 - p^2}}{p}$ **OR/*OF*** $\cos^2 36° = 1 - \sin^2 36°$ $\cos 36° = \sqrt{1 - (1 - p^2)}$ $\qquad = p$ $\tan 36° = \dfrac{\sin 36°}{\cos 36°}$ $\qquad = \dfrac{\sqrt{1 - p^2}}{p}$ 	✓ method ✓ value of p ✓ answer (3) ✓ method ✓ $\cos 36° = p$ ✓ answer (3)

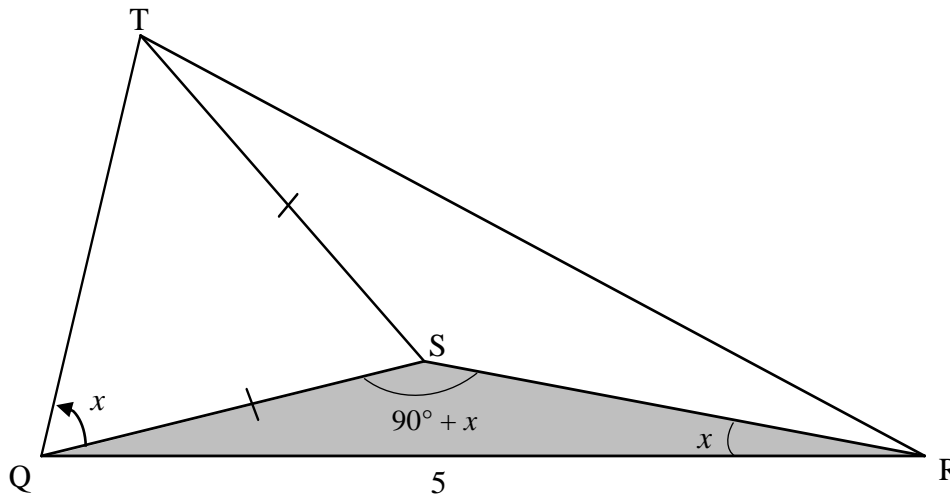| 5.3.2 | $\cos 108°$
 $= -\cos 72°$
 $= -\cos (2 \times 36°)$
 $= -(2\cos^2 36° - 1)$
 $= -2p^2 + 1$

 OR/*OF*
 $\cos 108°$
 $= -\cos 72°$
 $= -\cos (2 \times 36°)$
 $= -(1 - 2\sin^2 36°)$
 $= -1 + 2\left(\sqrt{1 - p^2}\right)^2$
 $= -1 + 2(1 - p^2)$
 $= -2p^2 + 1$

 OR/*OF*
 $\cos 108°$
 $= -\cos 72°$
 $= -\cos (2 \times 36°)$
 $= -(\cos^2 36° - \sin^2 36°)$
 $= -\left(p^2 - \left(\sqrt{1 - p^2}\right)^2 \right)$
 $= -\left(p^2 - (1 - p^2) \right)$
 $= -2p^2 + 1$

 OR/*OF*
 $\cos 108°$
 $= \cos(2 \times 54°)$
 $= 2\cos^2 54° - 1$
 $= 2(1 - p^2) - 1$
 $= 1 - 2p^2$
 OR/*OF*

 $\cos 108° = \cos(72° + 36°)$
 $\quad = \cos 72° \cos 36° - \sin 72° \sin 36°$
 $\quad = (2\cos^2 36° - 1)\cos 36° - (2\sin 36° \cos 36°)\sin 36°$
 $\quad = 2\cos^3 36° - \cos 36° - 2\cos 36° \sin^2 36°$
 $\quad = 2p^3 - p - 2p\left(\sqrt{1 - p^2}\right)^2$
 $\quad = 2p^3 - p - 2p + 2p^3$
 $\quad = 4p^3 - 3p$ | ✓ reduction
 ✓ double angle

 ✓ expansion
 ✓ answer i. t. o. p (4)

 ✓ reduction
 ✓ double angle

 ✓ expansion

 ✓ answer i. t. o. p

               (4)

 ✓ reduction
 ✓ double angle

 ✓ expansion

 ✓ answer i. t. o. p

               (4)

 ✓ double angle
 ✓✓ expansion

 ✓ answer i. t. o. p
               (4)

 ✓ expansion

 ✓ both double angle identities

 ✓ value of sin 36°
 ✓ answer i. t. o. p
               (4) |
| | | **[17]** |

QUESTION/*VRAAG* 6

6.1.1	$\cos(\alpha + \beta)$ $= \cos(\alpha - (-\beta))$ $= \cos \alpha \cos(-\beta) + \sin \alpha \sin(-\beta)$ $= \cos \alpha \cos \beta + \sin \alpha (-\sin \beta)$ $= \cos \alpha \cos \beta - \sin \alpha \sin \beta$	✓ $\cos(\alpha - (-\beta))$ ✓ expansion ✓ reduction (3)
6.1.2	$2 \cos 6x \cos 4x - \cos 10x + 2 \sin^2 x$ $= 2 \cos 6x \cos 4x - \cos(6x + 4x) + 2 \sin^2 x$ $= 2 \cos 6x \cos 4x - (\cos 6x \cos 4x - \sin 6x \sin 4x) + 2 \sin^2 x$ $= \cos 6x \cos 4x + \sin 6x \sin 4x + 2 \sin^2 x$ $= \cos 2x + 2 \sin^2 x$ $= 1 - 2 \sin^2 x + 2 \sin^2 x$ $= 1$	✓ $\cos 10x = \cos(6x + 4x)$ ✓ expansion of $\cos(6x + 4x)$ ✓ $\cos 2x$ ✓ $1 - 2 \sin^2 x$ ✓ answer (5)
6.2	$\tan x = 2 \sin 2x$ $\dfrac{\sin x}{\cos x} = 2(2 \sin x \cos x)$ $\sin x = 4 \sin x \cos^2 x$ $4 \sin x \cos^2 x - \sin x = 0$ $\sin x (4 \cos^2 x - 1) = 0$ $\sin x = 0 \qquad\qquad$ or $\qquad\qquad \cos^2 x = \dfrac{1}{4}$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad \cos x = -\dfrac{1}{2}$ $x = 180° + k.360°; \ k \in Z \quad$ or $\quad x = 120° + k.360°; \ k \in Z$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad x = 240° + k.360°; \ k \in Z$ **OR/*OF*** $\tan x = 2 \sin 2x$ $\dfrac{\sin x}{\cos x} = 4 \sin x \cos x$ $\sin x = 4 \sin x \cos^2 x$ $4 \sin x \cos^2 x - \sin x = 0$ $4 \sin x (1 - \sin^2 x) - \sin x = 0$ $3 \sin x - 4 \sin^3 x = 0$ $\sin x (3 - 4 \sin^2 x) = 0$ $\sin x = 0 \quad$ or $\quad \sin^2 x = \dfrac{3}{4}$ $\qquad\qquad \sin x = \dfrac{\sqrt{3}}{2} \quad$ or $\sin x = -\dfrac{\sqrt{3}}{2}$ $x = 180° + k.360°, k \in Z \ $ or $\ x = 120° + k.360°, k \in Z$ $\qquad\qquad\qquad$ or $x = 240° + k.360°, k \in Z$	✓ quotient identity ✓ double angle identity ✓ factors ✓ both equations ✓ $x = 180°$ ✓ $x = 120° \& 240°$ **OR/*OF*** $\quad x = \pm 120°$ ✓ $k.360°; \ k \in Z$ (7) ✓ quotient identity ✓ identity ✓ factors ✓ both equations ✓ $x = 180°$ ✓ $x = 120° \& 240°$ **OR/*OF*** $\quad x = \pm 120°$ ✓ $k.360°; \ k \in Z$ (7)
		[15]

QUESTION/*VRAAG* 7

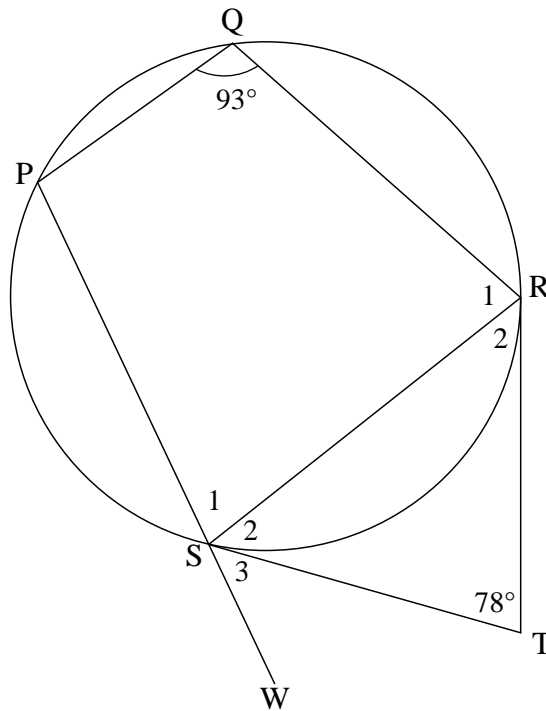| 7.1		✓ both turning points ✓ both *x* intercepts (−30° & 150°) ✓ shape (3)
7.2	Period = 120°	✓✓ answer (2)
7.3	$x = -30°$	✓ answer (1)
7.4	Range of/*waardeversameling van g*: $y \in [-1 ; 1]$ Range of/*Waardeversameling van* $\frac{1}{2}g$: $y \in \left[-\frac{1}{2} ; \frac{1}{2}\right]$ Range of/*Waardeversameling van* $\frac{1}{2}g + 1$: $y \in \left[\frac{1}{2} ; \frac{3}{2}\right]$ **OR/*OF*** Range of/*Waardeversameling van* $\frac{1}{2}g + 1$: $\frac{1}{2} \le y \le \frac{3}{2}$	 ✓ critical values ✓ correct notation (2) ✓ critical values ✓ correct notation (2)
		[8]

QUESTION/*VRAAG* 8

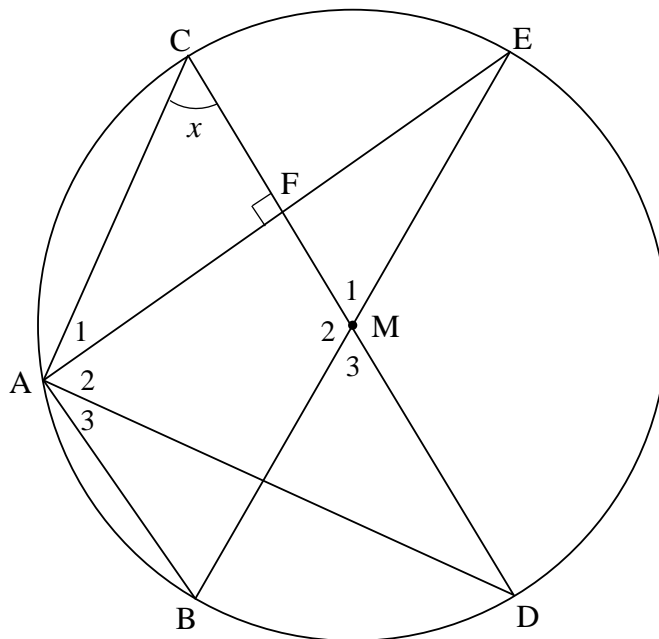| 8.1	In ΔSQR: $$\frac{QS}{\sin x} = \frac{QR}{\sin(90° + x)}$$ $$\frac{QS}{\sin x} = \frac{5}{\cos x}$$ $$QS = \frac{5\sin x}{\cos x}$$ $$QS = 5\tan x$$	✓ correct use of sine rule ✓ $\sin(90° + x) = \cos x$ ✓ $QS = \dfrac{5\sin x}{\cos x}$ (3)
8.2	$$\frac{QT}{\sin(180° - 2x)} = \frac{TS}{\sin x}$$ $$\frac{QT}{\sin 2x} = \frac{5\tan x}{\sin x}$$ $$QT = \frac{5\tan x \sin 2x}{\sin x}$$ $$QT = \frac{5\left(\dfrac{\sin x}{\cos x}\right)(2\sin x \cos x)}{\sin x}$$ $$QT = \frac{5\sin x\,(2\sin x)}{\sin x}$$ $$QT = 10\sin x$$	✓ correct use of sine rule ✓ $TS = QS = 5\tan x$ ✓ $QT = \dfrac{5\tan x \sin 2x}{\sin x}$ ✓ $\tan x = \dfrac{\sin x}{\cos x}$ ✓ $\sin 2x = 2\sin x \cos x$ (5)

OR/*OF*

$QT^2 = QS^2 + TS^2 - 2QS.TS\cos Q\hat{S}T$

$QT^2 = (5\tan x)^2 + (5\tan x)^2 - 2(5\tan x).(5\tan x)\cos(180° - 2x)$

$QT^2 = 50\tan^2 x - 50\tan^2 x(-\cos 2x)$

$QT^2 = 50\tan^2 x(1+\cos 2x)$

$QT^2 = 50\tan^2 x(1+2\cos^2 x - 1)$

$QT^2 = 50\tan^2 x(2\cos^2 x)$

$QT^2 = 100\dfrac{\sin^2 x}{\cos^2 x}(\cos^2 x)$

$QT^2 = 100\sin^2 x$

$QT = 10\sin x$

✓ correct use of cos rule
✓ TS = QS = 5tan*x*

✓ $\cos 2x = 2\cos^2 x - 1$ & reduction

✓ $\tan x = \dfrac{\sin x}{\cos x}$
✓ $QT^2 = 100\sin^2 x$

(5)

OR/*OF*

$TS^2 = QS^2 + TQ^2 - 2QS.TQ.\cos x$

$(5\tan x)^2 = (5\tan x)^2 + TQ^2 - 2(5\tan x).TQ.\cos x$

$0 = TQ^2 - 2(5\tan x).TQ.\cos x$

$0 = TQ[TQ - 10\tan x.\cos x]$

$TQ = 10\tan x.\cos x \quad (TQ \neq 0)$

$= 10\dfrac{\sin x}{\cos x}.\cos x$

$= 10\sin x$

✓ correct use of cos rule
✓ TS = QS = 5tan*x*
✓ quadratic equation ito TQ

✓ TQ = 10tan *x* . cos *x*
✓ $\tan x = \dfrac{\sin x}{\cos x}$

(5)

| 8.3 | Area of $\triangle TQR = \dfrac{1}{2}.TQ.QR\sin T\hat{Q}R$ $= \dfrac{1}{2}(10\sin 25°)(5)(\sin 70°)$ $= 9{,}93\,\text{unit}^2$ | ✓ correct substitution into the area rule ✓ answer (2) |

[10]

QUESTION/VRAAG 9

9.1	tangents from same(common) point/*raaklyne vanaf dieselfde punt*	✓ R (1)
9.2.1	$\hat{S}_2 = S\hat{R}T$ [∠s opp equal sides/*∠e teenoor gelyke sye*] $\therefore \hat{S}_2 = 51°$ [sum of ∠s in Δ/*som van ∠e in Δ*]	✓ R ✓ S (2)
9.2.2	$\hat{S}_2 + \hat{S}_3 = 93°$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $\hat{S}_3 = 42°$ **OR/*OF*** $\hat{S}_1 = 87°$ [opp ∠s of cyclic quad/*teenoorst ∠e v kdvh*] $\hat{S}_3 = 180° - (87° + 51°)$ $\hat{S}_3 = 42°$ [∠s on a str line/*∠e op reguitlyn*]	✓ R ✓ answer (2) ✓ R ✓ answer (2)
		[5]

QUESTION/*VRAAG* 10

10.1	line from centre $\perp$ to chord/*lyn vanaf middelpunt $\perp$ op koord*	✓ R
		(1)
10.2	$\therefore \hat{A}_1 = 90° - x$ [sum of $\angle$s in Δ/*som van $\angle$e in Δ*]	✓ S
	$\therefore \hat{M}_1 = 180° - 2x$ [$\angle$at centre=2× at circumf/*midpts$\angle$= 2×omtreks$\angle$*]	✓ S ✓ R
		(3)
10.3	$\hat{CAD} = 90°$ [$\angle$ in semi circle/*$\angle$ in halfsirkel*]	✓ S ✓ R
	$\hat{A}_2 = 90° - (90° - x)$	
	$\hat{A}_2 = x$	✓ S
	$\therefore \hat{A}_2 = \hat{C} = x$	
	$\therefore$ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*]	✓ R
		(4)
	OR/*OF*	
	$\hat{EMD} = 2x$ [adj suppl $\angle$s/*aanligg suppl $\angle$e*]	✓ S
	$\therefore \hat{A}_2 = x$ [$\angle$at centre= 2×$\angle$ at circumf/*midpts$\angle$= 2 ×omtreks$\angle$*]	
	$\therefore \hat{A}_2 = C = x$	✓ S ✓ R
	$\therefore$ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*]	✓ R
		(4)
	OR/*OF*	
	$\hat{M}_3 = 180° - 2x$ [vert. opp/ *regoorstaande $\angle$e*]	
	$\therefore \hat{A}_3 = 90° - x$ [$\angle$at centre= 2×$\angle$ at circumf/*midpts$\angle$= 2 ×omtreks$\angle$*]	✓ S
	$\hat{BAE} = 90°$ [$\angle$ in semi-circle /*$\angle$ in halfsirkel*]	✓ R
	$\therefore \hat{A}_2 = C = x$	✓ S
	$\therefore$ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*]	✓ R
	OR/*OF*	(4)

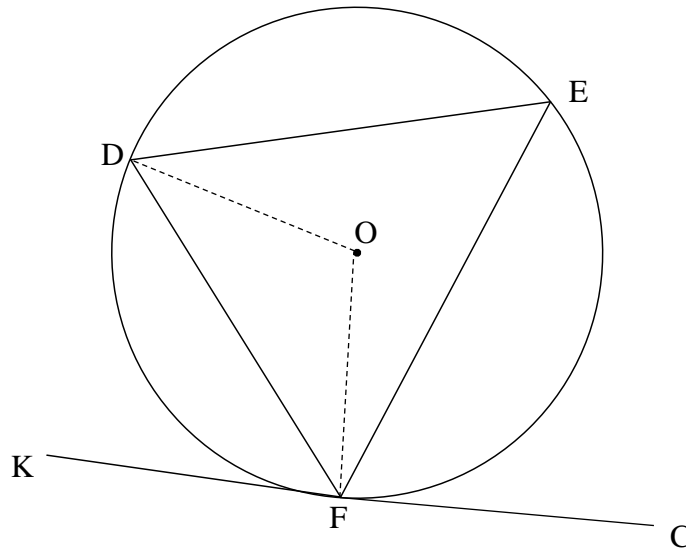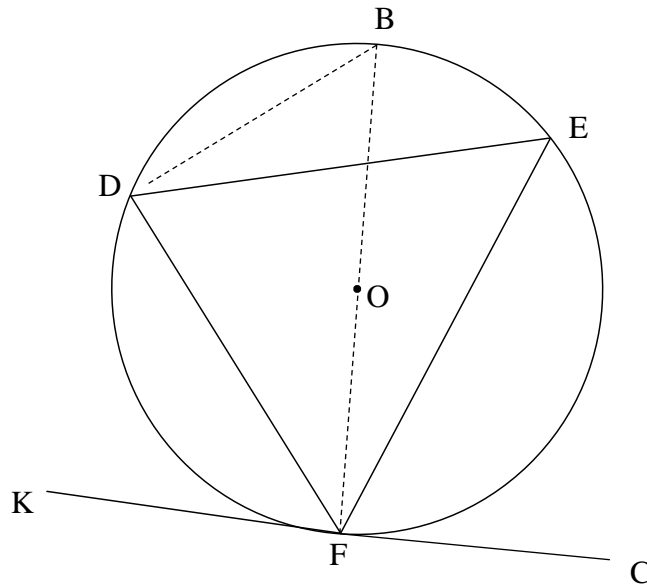|		
	CD ∥ AB [midpt. Thm/ *middelpuntst.*] $B\hat{A}E = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $\therefore \hat{A}_3 = \hat{D} = 90° - x$ [alt.∠s; CD∥AB/*verwiss ∠e*] $\therefore \hat{A}_2 = x = $ C $\therefore$ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $C\hat{A}D = 90°$ [∠ in semi circle/∠ *in halfsirkel*] AC = diameter [converse ∠ in semi circle/*omgek ∠ in halfsirkel*] $\therefore$ AD is a tangent [converse radius ⊥ tangent/*omgek radius⊥rkl*]	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ S ✓ R (4)

10.4	AF = FE and BM = ME [given & radii]
	$\therefore$ FM = $\frac{1}{2}$ AB = 12 units [Midpt Theorem/*middelptstelling*]
	EM = MB = CM = 18 units [radii]
	$\therefore$ EB = 36 units [diameter = 2 radius]
	$\therefore AE^2 = (36)^2 - (24)^2$ [Pythagoras]
	   AE = $12\sqrt{5}$ or 26,83 units
	
	OR/*OF*
	
	AF = FE and BM = ME [given & radii]
	$\therefore$ FM = $\frac{1}{2}$ AB = 12 units [Midpt Theorem/*middelpuntstelling*]
	EM = MB = CM = 18 units [radii]
	$\therefore FE^2 = (18)^2 - (12)^2$ [Pythagoras]
	   FE = $6\sqrt{5}$
	    AE = $12\sqrt{5}$ or 26,83 units
		✓ FM = 12 ✓ R
		
		✓ EB = 36
		✓ using Pyth correctly
		✓ answer
		(5)
		
		✓ FM = 12 ✓ R
		
		✓ EM = 18
		✓ using Pyth correctly
		
		✓ answer
		(5)

		[13]

QUESTION/*VRAAG* 11

11.1

	Construction: Draw diameter BF and draw BE *Konstruksie: Trek middellyn* BF *en verbind* BE	✓Constr
	$B\hat{F}K = 90°$ or $D\hat{F}K = 90° - B\hat{F}D$ [radius ⊥ tangent/*raaklyn*]	✓ S ✓ R
	$B\hat{E}F = 90°$ [∠ in semi-circle/*semi-sirkel*]	✓ S
	$\therefore D\hat{E}F = 90° - B\hat{E}D$	
	$= 90° - B\hat{F}D$ [∠s same segment/∠*e dieselfde segment*]	✓ S/R
	$\therefore D\hat{F}K = D\hat{E}F$	
		(5)

OR/*OF*

	Construction: Draw radii DO and OF	✓ construction
	Konstruksie: Trek radii DO en OF	
	$O\hat{F}K = 90°$ or $D\hat{F}K = 90° - O\hat{F}D$ radius⊥ tangent/*raaklyn*]	✓ S ✓R
	$O\hat{D}F = O\hat{F}D$ [∠s opp = sides/*∠e teenoor = sye*]	✓ S
	∴ $D\hat{O}F = 180° - 2O\hat{F}D$ [∠s of Δ/*∠e van Δ*]	
	$D\hat{E}F = 90° - O\hat{F}D$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*]	✓ S/R
	∴ $D\hat{F}K = D\hat{E}F$	
		(5)

OR/OF

	Construction: Draw diameter BF and join BD.	✓ construction
	Konstruksie: Trek middellyn BF en verbind BD.	
	$B\hat{F}K = 90°$ or $D\hat{F}K = 90° - B\hat{F}D$ [radius⊥ tangent/*raaklyn*]	✓ S ✓/R
	$F\hat{D}B = 90°$　　[∠ in half circle/*semi-sirkel*]	✓ S
	$\hat{B} = 90° - B\hat{F}D$	
	$\therefore D\hat{F}K = \hat{B}$	
	but $\hat{B} = \hat{E}$　　[∠s same segment/∠*e dieselfde segment*]	✓ S/R
	$\therefore D\hat{F}K = \hat{E}$	(5)

11.2

11.2.1(a)	$\hat{K}_4 = \hat{S}_1$ [tan chord theorem/*raaklynkoordstelling*] $\hat{M}_2 + \hat{M}_3 = \hat{S}_1$ [corresp ∠s; / *ooreenk* ∠*s;* MN ‖ KS] $\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = N\hat{M}L$	✓ S ✓ R ✓ S ✓ R (4)

11.2.1(b)	$\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = N\hat{M}L$
	 $\therefore$ KLMN is a cyclic quad [ext ∠ of quad = opp int ∠ /
	                         buite ∠ *van vh = teenoorst binne* ∠]
	
	 OR/OF
	
	 $N_1 = \hat{K}_1 + \hat{K}_2 = N\hat{K}S$ [corresp ∠s; / *ooreenk* ∠*s;* MN ‖ KS]
	 $N\hat{K}S = K\hat{L}S$ [tan chord theorem / *raaklynkoordstelling*]
	 $\hat{N}_1 = K\hat{L}S$
	 $\therefore$ KLMN is a cyclic quad [ext ∠ of quad = opp int ∠ /
	                         buite ∠ *van vh = teenoorst binne* ∠]
	
	 OR/OF
		✓ R
		
		 (1)
		
		 ✓ R
		
		 (1)

	$NKL = 180° - K_4$ [adj. suppl.] $\therefore NKL = 180° - NML$ [proved] $\therefore$ KLMN is a cyclic quad [opp.$\angle$s supplementary]	✓ R (1)
11.2.2	In $\triangle LKN \| \triangle KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$s in the same seg / $\angle e$ in dieselfde sirkel segm] $\hat{L}_1 = \hat{M}_2$ [$\angle$s in the same seg / $\angle e$ in dieselfde sirkel segm] $\;\;\;= \hat{K}_2$ [alt $\angle$s; / verw$\angle e$; MN|KS] $N\hat{K}L = M\hat{S}K$ [$\angle$s of $\triangle$ / $\angle e$ van $\varDelta$] $\triangle LKN \| \triangle KSM$ **OR/OF** In $\triangle LKN \| \triangle KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$s in the same seg / $\angle e$ in dieselfde sirkel segm] $N\hat{K}L = \hat{M}_1$ [ext $\angle$ of cyclic quad/*buite$\angle$ van koordevh*] $\;\;\;= \hat{S}_2$ [corresp $\angle$s/*ooreenk $\angle e$*; KS | NM] $\triangle LKN \| \triangle KSM$ [$\angle,\angle,\angle$] **OR/OF** In $\triangle LKN \| \triangle KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$s in the same seg / $\angle e$ in dieselfde sirkel segm] $\hat{K}_4 + N\hat{K}L = \hat{S}_1 + \hat{S}_2$ [$\angle$s on straight line/$\angle e$ op reguitlyn] $\therefore N\hat{K}L = \hat{S}_2$ [$\hat{K}_4 = \hat{S}_1$] $\triangle LKN \| \triangle KSM$ [$\angle,\angle,\angle$]	✓ S ✓ R ✓ S ✓ S/R ✓ S (5) ✓ S ✓ R ✓ S/R ✓ S ✓ R (5) ✓ S ✓ R ✓ S/R ✓ S ✓ R (5)
11.2.3	$\dfrac{LK}{KS} = \dfrac{KN}{SM}$ [$\triangle LKN \| \triangle KSM$] $\therefore \dfrac{12}{KS} = \dfrac{4}{3}$ $KS = 9$ units	✓ S ✓ R ✓ substitution ✓ answer (4)
11.2.4	$4SM = 3KN$ $SM = \dfrac{3(8)}{4}$ $SM = 6$ $\dfrac{LT}{NL} = \dfrac{LS}{ML}$ [line | one side of $\triangle$ / *lyn | een sy v $\triangle$*] $\dfrac{LT}{16} = \dfrac{13}{19}$ $LT = \dfrac{208}{19} = 10{,}95$	✓ SM = 6 ✓ S ✓ R ✓ answer (4)
		[23]

TOTAL/*TOTAAL*: 150